Differential Pathways of Learning: How Four Low-Achieving Undergraduate Students used a Graphics Calculator

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This paper reports on a year-long technology-rich teaching experiment conducted with four low-achieving undergraduate students. Using questionnaires, classroom assessment tasks, self-report journals, interviews and field notes, the study identified three stages of cognitive development – *computational*, *technician*, and *multirepresentational* – which reflect successively higher levels of tool-based engagement and performance. Four tool-based pathways or "journeys" were also identified, illustrating how these mathematically weak students navigated their way through the stages of the model. They are: a restricted learning pathway, a linear learning pathway, a non-linear learning pathway, and multiple learning pathways.

The past two decades has witnessed substantial increases in the number of students, particularly low-achieving ones, undertaking technology-rich mathematics-based undergraduate degrees for which they are poorly prepared (Hillel et al., 1992). Secondary schools and to a lesser extent post-secondary institutions have been keen to include technology into their learning programs, the basic assumption being that the integration of tools such as Computer Algebra Systems and graphics calculators has the potential to better assist students with their learning, and are superior to the more traditional methods of course delivery. Furthermore, the majority of studies that have reported these findings have been carried out with average to high-achieving students. However, little research has been done with low achievers, particularly at the post-secondary level (Dunham & Dick, 1994).

I began integrating technology-rich activities into a mathematics "bridging" subject in the mid-1990s with low-achieving students who were embarking on engineering, business and education degrees. Most students in my charge were poorly prepared to embark on courses that required a thorough grounding in Year 12 mathematics. The courses in which I was involved prior to the study were typical of many undergraduate programs found in Australian universities at that time: syllabuses were content laden, teaching strategies emphasised information transmission as a means of course delivery, and assessment practices valued the importance of mid-semester tests and end-of-semester examinations as measures of performance. Consequently, classroom learning remained a passive process of knowledge and skill acquisition with the attainment of high grades as indicators of success (Confrey, 1990).

The Study

This paper reports on the final of three technology-rich teaching experiments that were conducted over a period of three academic years (Lindsay, 2002). The developmental models of Pirie and Kieren (1994) and the SOLO Taxonomy (Biggs & Collis, 1982), together with a constructivist view of learning mathematics (Cobb, Yackel & Wood, 1990) provided the cognitive and pedagogical orientation for the study. The evidence that emerged from the first two phases of the study suggested that low-achieving students were

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not readily embracing the technology as was originally expected. In fact, quite the contrary: many avoided using the tool and when they did engage with it they used it incorrectly and/or inappropriately. The findings that emerged from these teaching experiments informed and provided a direction for the final teaching experiment. Using the graphics calculator as the preferred technology, and conducted over a period of two semesters, it consisted of the detailed quantitative and qualitative analyses of ten students' data sets. The data collection instruments included questionnaires, self-report journals, interviews, field notes and classroom assessment tasks. The latter were evaluated using a 5-level performance-based scoring rubric – developed in an earlier phase of the study – that provided a clearer picture of the qualitatively different ways in which students were engaging with tool-based tasks (Lindsay, 1999, 2000). The remainder of this paper will focus on the cognitive development of, and learning pathways taken by four of these students as they responded to assessment tasks that were given to them during this final phase.

The Four Case Studies

Dean: Computational User; Restricted Learning Pathway

Dean (age 19) was case-studied because he performed poorly in his entry-level test, and he had not used a graphics calculator before. His entry-level test responses revealed that both his graphing and basic algebraic skills were very weak. Assessment tasks administered to students over the course of the year (as will be seen later) relied predominantly on tool-based strategies although supporting pencil-and-paper activities were considered important components as well. By year's-end Dean had made limited progress and his graphics calculator (ninth) and pencil-and-paper (ninth) rankings at the end of each semester, relative to the cohort of ten students, were consistently low. His tool-based responses suggested that he was operating at a very basic level with respect to his procedural and conceptual knowledge (Hiebert & Carpenter, 1984). The *computational* stage to which Dean's responses were classified, is characterised by the following rubric indicators of performance:

- Output is being copied regardless of its accuracy or relevance
- Limited or no progress towards completion of the task, or no response to the question at all
- Incorrect and inaccurate use of mathematics
- Mathematics explanations brief or absent; limited technical ability in using the graphics calculator
- Employs graphics calculator-generated solutions instead of algebraically derived solutions
- No evidence of correct use of mathematics language, symbols and conventions
- No links between graphical, symbolic and numerical representations
- Makes no attempt at moving to generalisations.

For instance, Dean's response to a trigonometry question that required him to plot and describe the graphs of $f(x) = \cos x$ and $g(x) = 3 \cos (x/2)$, including the location of amplitudes, axial intercepts and periods is shown below in Figure 1.

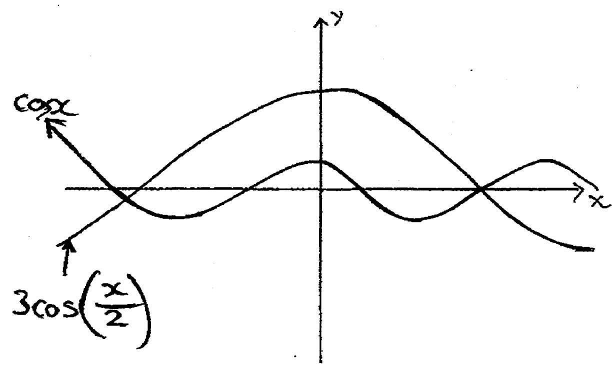

Figure 1. Dean's incomplete response to a trigonometry question.

Dean's graph drawing lacked all detail, the graphics calculator output being copied with little regard given to the properties of the images being generated by the calculator. He appeared to be using the tool as a "button-pushing" device. When asked to compare the functions' periods he said this:

> *g(x)* doesn't look like a zigzag. It's more like an oval mountain shape and is much higher than *f(x)*. *f(x)* is a more spread out graph and is a zigzag curve shape.

Computational users display limited use of mathematical language which restricts their ability to explain graph plots. Progress with tool-based tasks appears to be contingent upon their possesson of appropriate pencil-and-paper skills and knowledge. Their learning pathways remain restricted, and it is suggested that only by continually moving back-and-forth – or "folding back" (Pirie & Kieren, 1994) – between their newly acquired pencil-and-paper (P/P) skills and knowledge and their existing tool-based skills and knowledge (shown in Figure 2 below by a circular broken line) will students be able to achieve tool-related success at the most basic level.

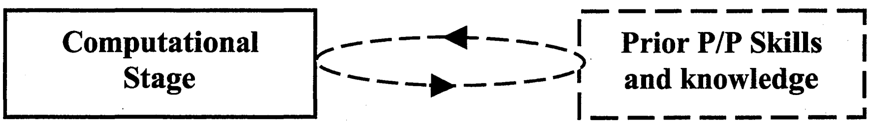

Figure 2. The restricted learning pathway of Dean, a *computational* user.

Stuart: Technician; Linear Learning Pathway

Stuart (age 17) was selected for the study on the basis of three attributes: he was not a prior user of the graphics calculator, he performed poorly in his entry-level test, and he displayed an unusually high level of interest in the graphics calculator early on in the course. His graphics calculator ranking within the sample of ten students improved (from sixth to fourth) whilst his pencil-and-paper ranking dropped from sixth to seventh between the end of Semester 1 and the end of Semester 2.

Stuart's enthusiasm for the graphics calculator was evident in his interview responses. For instance, his successful sketch of $f(x)=2x^3+10x^2-2x+50$ early in the year is shown in Figure 3 below.

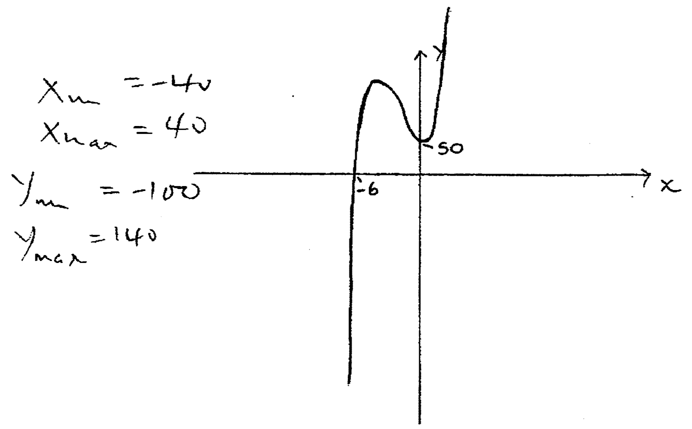

Figure 3. Stuart's technical proficiency in sketching a cubic function.

Stuart's proficiency in carefully manipulating the "windows" key is illustrated above. The following interview excerpt described how he achieved this graph plot:

> Yeah, I just used the graphics calculator and the "y = ", pressed the "y = " and then just typed in the y ... just graphed that using the "windows", and then just used the "trace" key to find where it touched the X-intercept and the Y-intercept, the Y (intercept) is 50, and the negative … is 5.89, and … just used the "trace" key again to go to one of the turning points which is about (-3.40, 93.79). (Interview 1, Semester 1)

Technicians, to which Stuart's responses are typical, are characterised by the following indicators of performance:

- Makes some progress towards completing the task
- Makes some progress towards correctly and accurately using mathematics
- Limited, though purposeful explanations and interpretations
- Is usually technically competent in using a graphics calculator
- Uses graphics calculator-generated solutions instead of algebraically derived solutions
- Some mathematics language, symbols and conventions used correctly
- Partial but limited links between graphical, symbolic and numerical representations
- Unable to move to generalisations

Technicians have more control of and familiarity with the technical aspects of the tool, employing it as a scaffolding device to build understanding in new ways (Kutzler, 1997). Drawing on the *computational* stage partial links are evident between graphical, symbolic and numerical representations and the graphics calculator is being used – with limited success – to explain and interpret screen images such as graph-plots. This is the stage at which students are trying, for the first time, to engage in an intellectual partnership with the tool (Salomon et al., 1991). As with *computational* users, gaps in *technicians'* pencil-and-paper skills and knowledge appear to compel them to use the tool more as the means by which they can achieve success in their mathematics tasks. The learning pathway taken by Stuart is linear: it begins at the *computational* stage and progresses to the *technician* stage, with brief task-based learning episodes suggesting that he was beginning to engage in *multirepresentational* thinking (see broken-lined unidirectional arrow in Figure 4 below). Throughout, Stuart "folded back" at appropriate times to retrieve pencil-and-paper skills and knowledge to support and enrich his tool-based learning.

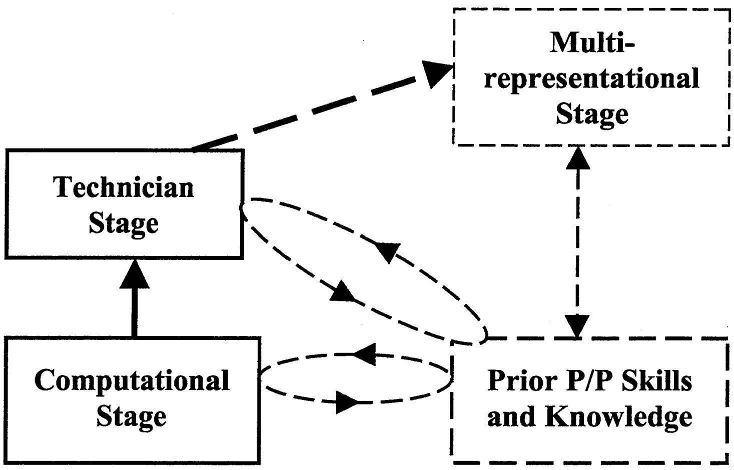

Figure 4. The linear learning pathway taken by Stuart, a *technician,* who was beginning to engage in *multirepresentational* thinking (see broken-lined, unidirectional arrow).

Lois: Multirepresentational User; Multiple Learning Pathways

Lois (age 18) was case studied because she had studied the more advanced Year 12 "Maths Methods" subject and was thus more mathematically experienced than her fellow students; and secondly, she was a prior graphics calculator user. Her pencil-and-paper ranking relative to the cohort of ten students (end-of-Semester 1: second; end-of-Semester 2: first) remained high whilst her graphics calculator ranking fell from second in Semester 1 to sixth in Semester 2.

Lois had few difficulties with tool-based tasks. For instance, her response to the task shown in Figure 5 demonstrates that she was technically proficient in using the graphics calculator and was correctly linking symbolic and graphical representations.

The responses of *multirepresentational* users such as Lois are characterised by the following performance indicators:

- Substantial progress towards completion of tasks
- Substantial progress towards the correct and accurate use of mathematics
- Substantial progress towards full and detailed explanations and interpretations
- Is generally technically competent in using the graphics calculator
- Prefers graphics calculator-generated solutions to algebraically derived solutions
- May or may not use the graphics calculator to check algebraically derived solutions
- Substantial progress towards correctly linking graphical, symbolic and numerical representations
- Has difficulty moving to generalisations

Whilst *technicians* are working with images, users at the *multirepresentational* stage are attempting to extract meaning from the properties of these images. Even though Lois engaged successfully with the graphics calculator she preferred, wherever possible, to employ pencil-and-paper strategies. When asked why, she said this:

> I just think of it as an aid, like a tool to help us through some problems and to verify it … putting it on the calculator you are not achieving anything, you have to know why it is like that, so when you showed me that maximum height reached (a non-routine problem), I am just not satisfied with

getting the answer off the calculator, it is like cheating. You know, it is like I haven't really worked anything out. It is like the calculator has done all the work for me, like I am not really learning anything. (Interview 1, Semester 1)

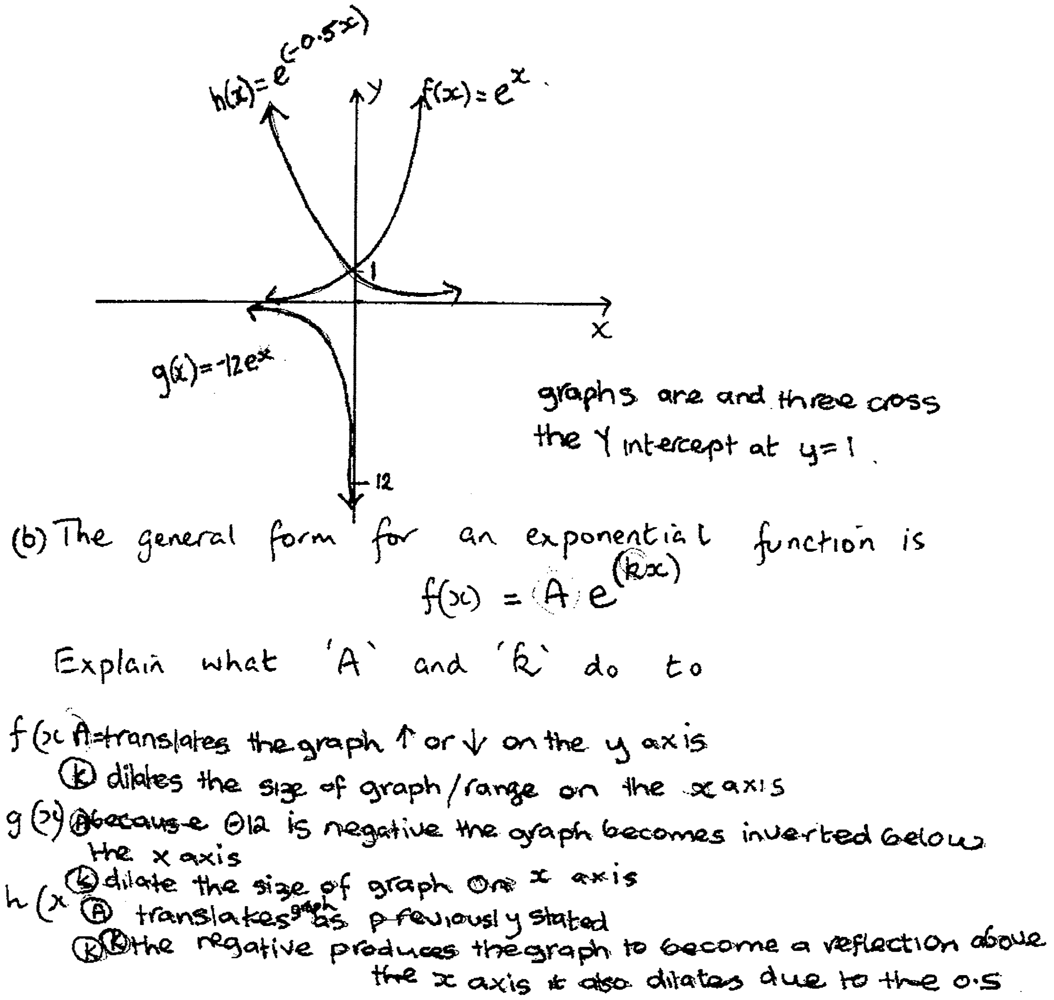

Figure 5. Lois' proficiency in answering a *multirepresentational* task.

In a later interview she again spoke about how the graphics calculator assisted her with her mathematics, but was still reluctant to use it other than for straightforward tasks such as graph plotting, the execution of computations and as a checking instrument:

> I think that basically that the calculator is there so you know that you are on the right track. You know, you can do everything by hand, but I think it is good so you know you have got another tool, so you make sure that you are on the right track and your graph is looking okay. If I didn't have the calculator I probably wouldn't be able to draw the graphs by hand any more, because I rely on it a lot. (Interview 3, Semester 2)

Lois searched for alternative pathways of learning to achieve competence at the *multirepresentational* stage. For instance, tasks that compelled her to use the tool resulted in a the linear pathway: *computational → technician → multirepresentational*. However, if the task could be accomplished with minimal use of the graphics calculator she employed a *computational → multirepresentational* pathway, bypassing the *technician* stage (see Figure 6).

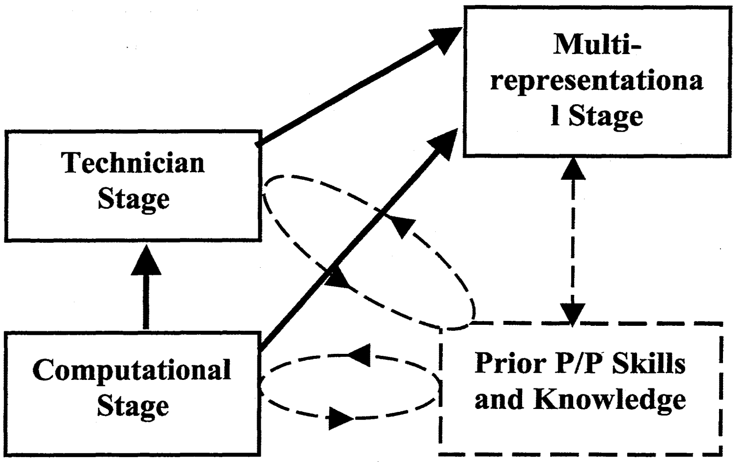

Figure 6. The multiple learning pathways taken by Lois, a *multirepresentational* user.

Nicky: Multirepresentational User; Non-Linear Learning Pathway

Nicky (age 23) was chosen for the study for three reasons: she was a mature-age student, she performed poorly in her entry-level test, and she had not used a graphics calculator prior to commencing her university studies. Like Lois, her task responses suggested that she was a *multirepresentational* user. Her search for meaningful links between the graphics calculator and her knowledge of mathematics, together with her views concerning the graphics calculator, are illustrated in the following comments:

> I am not learning the maths deeply. I feel that I need more exercises so that I can develop a more confident memory of the processes. I feel I am rote learning ... Remembering! Deep learning all of the topics. I still feel that I am rote learning. (Self-report journal, Semester 1)

> This (graphics calculator) doesn't help me to remember. If I was to do everything on a calculator I wouldn't remember as well as I remember this because I actually find meaning in that, whereas I don't have any sense of meaning with this. Last night I just discovered out of the blue how to factorise and everyone was saying just have a look at it and you should be able to estimate, but I don't understand which sections that I was supposed to estimate or what I was supposed to use, until I had actual meaning of what the equation was I couldn't understand it and I think that is the same with the calculator, until I can actually understand exactly what the calculator is doing, until I can read the calculator, I don't think I will understand it. (Interview 3, Semester 2)

Nicky was not as proficient with the tool as Lois. Consequently, her learning pathway reflected tool avoidance wherever possible and resulted in non-linear journeys (see Figure 7). For instance, for some tasks she began at the *computational* stage, progressed to the *multrepresentational* stage, briefly returned to the *technician* stage, and finally moved back up to the *multirepresentational* stage again. There was no evidence to suggest that tool proficiency at the *computational* stage was necessarily assisting her at the *technician* stage (shown by a broken-lined box to indicate that her proficiency at this level was incomplete). Hence, a unidirectional arrow between these two stages is not present in Figure 7. She possessed the prerequisite pencil-and-paper skills and knowledge to operate successfully at the *multrepresentational* stage but appeared to be unable and unwilling to engage with the tool sufficiently to demonstrate proficiency at the *technician* stage.

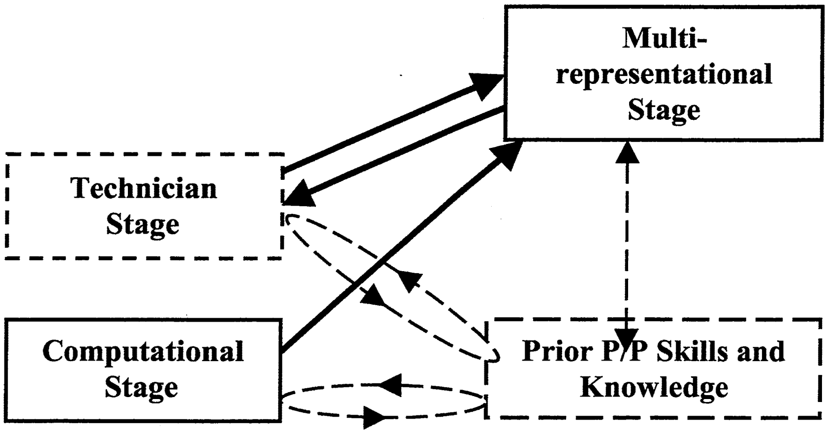

Figure 7. The non-linear learning pathway taken by Nicky, a *multirepresentational* user.

Discussion

An analysis of the tool-based assessment task responses of low-achieving students such as Dean, Stuart, Lois and Nicky revealed that they appeared to be operating at three levels of cognitive development. The lowermost level – the *computational* stage – represents the stage at which the user's engagement and performance with the tool is at its most primitive. This is followed by the *technician* and *multirepresentational* stages which depict increasing levels of successful user-tool engagement and performance respectively. The stages are qualitatively different and appear to be hierarchical, each characterised by indicators of performance.

For satisfactory performance at each stage students must possess prerequisite pencil-and-paper (P/P) skills and knowledge appropriate to that stage. This is represented diagrammatically in Figure 8 by the "Prior P/P Skills and Knowledge" box. The circular and bi-directional broken-lined arrows represent a dynamic interaction in which students are continually "folding back" (Pirie & Kieren, 1994) from the *computational*, *technician,* and *multirepresentational* boxes, to collect, reflect on, improve and reconstruct their prior skills and knowledge so that tool-based understanding can be supported, enhanced and enriched.

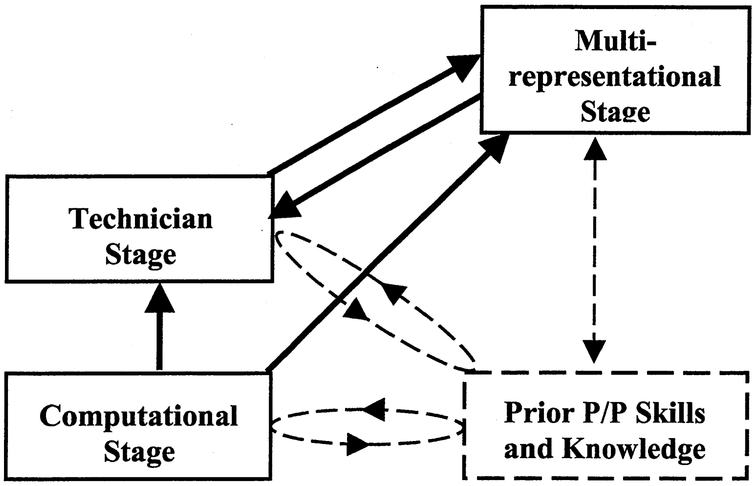

Figure 8. The three stages – *computational, technician and multirepresentational* – to which task responses for four low-achieving students were classified, and their differential pathways of learning.

Figure 8 also illustrates four tool-based pathways or journeys by which the four low-achieving students progressed as they navigated their way through successively higher stages of the model: a restricted learning pathway, a linear learning pathway, a non-linear learning pathway, and multiple learning pathways. The evidence presented here, along with the six other case studies conducted during the year, suggests that educators need to be far more aware of the qualitatively different ways in which low-achieving students engage with technology-rich tasks. While further research is needed to test the utility of the model, it is hoped that it will act as a powerful framework in assisting teachers understand and work with mathematically weak students as they engage with technological tools.

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